1.

a) $Ne\,(1s^2 2s^2 2p^6)$ $\qquad L=0 \quad S=0$

b) $E_B = Z_{eff}^2 \cdot 13.6 \Rightarrow Z_{eff} = \sqrt{\frac{21.56}{13.6}} = 1.26$

or $E_B = 13.6 \cdot \frac{Z_{eff}^2}{n^2} \Rightarrow Z_{eff} = n \cdot \sqrt{\frac{21.56}{13.6}} = 2.52$

c) $L = 1 \quad S = 0, 1 \Rightarrow J = 0, 1, 2$

d) $\lambda = 73.6\ nm \Rightarrow \Delta E = h \cdot \nu = \frac{hc}{\lambda} = \frac{6.6 \cdot 10^{-34} \cdot 3 \cdot 10^{8}}{73.6 \cdot 10^{-9}} \frac{1}{1.6 \cdot 10^{-19}\ C} = \underline{\underline{16.81\ eV}}$

$E_B = 21.56 - 16.81 = 4.75\ eV$

$= \frac{13.6}{(3 - \Delta n)^2} \Rightarrow 3 - \Delta n = \sqrt{\frac{13.6}{4.75}} = 1.69$

$\Longrightarrow \Delta n = 1.31$

e) $E_b(4s) = \frac{13.6}{(4 - 1.31)^2} = 1.88\ eV$

$\Rightarrow E_{ex} = 21.56 - 1.88 = 19.68\ eV \Rightarrow \lambda = \frac{hc}{\Delta E} = \frac{6.6 \cdot 10^{-34} \cdot 3 \cdot 10^{8}}{19.68 \cdot 1.6 \cdot 10^{-19}}$

$= 62.9\ nm$

2.

lution problem : <u>Zeeman slower</u>

) Metastable states cannot decay to lower lying states via radiative E1 transit.
He $(1s2s\ ^3S^1)$ is metastable because it is the lowest lying triplet state,

) absorption of a laserphoton transfers momentum $\hbar k$ to the atom slowing it down, momentum recoils following photon emission average out to zero.

) $v = v_0 - a \cdot t$ with $a = \frac{F}{M} = \frac{1}{M}\frac{dp}{dt} = \frac{h}{\lambda \tau M} \Rightarrow t_{stop} = \frac{v_0}{a} = \frac{v_0 \lambda \tau M}{h} = 1.09\ ms$

$L = v_0 t - \frac{1}{2} a t^2 = \frac{v_0^2}{2a} = \frac{v_0^2 \lambda \tau M}{2h} = 54.6\ cm$

d) $\Delta \nu_{1/2} = \frac{1}{2\pi\tau} = 1.59\ MHz$; Doppler shift: $\Delta \nu = \nu \cdot \frac{v_0}{c} = \frac{v_0}{\lambda} = 923\ MHz$

e) $z(t) = v_0 t - \frac{1}{2} a t^2 \Rightarrow t = \frac{v_0 \pm \sqrt{v_0^2 - 2az}}{a}$; $v(z) = v_0 - at = \sqrt{v_0^2 - 2az} = v_0 \sqrt{1 - \frac{z}{L}}$

Zeeman shift: $\Delta E_{Zeeman} = g_J \mu_B m_J B$; $g_J(\text{groundstate}) = 2$, $g_J(\text{exc. state}) = \frac{3}{2}$

$E_{ground} = 2\mu_B B$; $E_{exc.} = E_0 + 3\mu_B B \Rightarrow \Delta E_{Zeeman} = E_0 + \mu_B B$; $\Delta E_{Doppl} = E_0 (1 + \frac{v}{c})$

$\Delta E_{Zeeman} = \Delta E_{Doppl} \Rightarrow B(z) = \frac{E_0 v(z)}{\mu_B c} = \frac{h v_0}{\lambda \mu_B} \sqrt{1 - \frac{z}{L}}$

f) No, the $1s2p\ ^1P$ state decays also to the $1s^2\ ^1S$ groundstate.

(3) See Haken and Wolf problem 12.6

a) $V_{es} = -\vec{\mu}_s \vec{B}_e = g_s \mu_B \sqrt{s(s+1)}\, B_e \cdot \cos(\vec{s}, \vec{e})$

$$= g_s \mu_B \sqrt{s(s+1)}\, B_e \cdot \frac{[j(j+1)-e(e+1)-s(s+1)]}{2\sqrt{s(s+1)}\sqrt{e(e+1)}} \equiv \frac{a}{2}[j(j+1)-e(e+1)-s(s+1)]$$

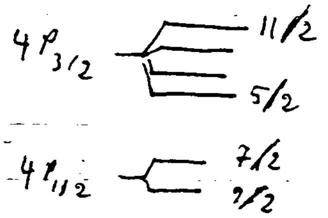
$$\Rightarrow B_e = \frac{a}{2}\sqrt{e(e+1)} \cdot \frac{1}{\mu_B}$$

b) $\Delta E = hc\left(\frac{1}{\lambda_1}-\frac{1}{\lambda_2}\right) = h\cdot c\; 10^9\left(\frac{1}{766.7}-\frac{1}{770.1}\right) = 1.14\cdot 10^{-21}\, J$

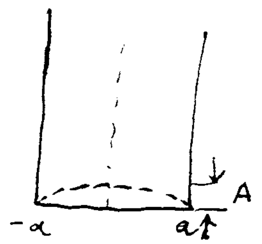
$$B_e = \frac{1.14\cdot 10^{-21}}{2}\sqrt{1.2}\cdot\frac{1}{\mu_B} = 86.9\, T$$

c) $4P_{1/2}: J = \frac{1}{2},\ I = 4 \Rightarrow F = \frac{7}{2}$ and $F = \frac{9}{2}$

$4P_{3/2}: J = \frac{3}{2}\ \ I = 4 \Rightarrow F = \frac{5}{2}, \frac{7}{2}, \frac{9}{2}, \frac{11}{2}$



(4) a) $\mathcal{H}\psi = -\frac{\hbar^2}{2m}\psi'' = E\psi$



$u_0 = \frac{1}{\sqrt{a}}\cos\frac{\pi x}{2a} \qquad u_1 = \frac{1}{\sqrt{a}}\sin\frac{\pi x}{a}$

b) $E_0 = \frac{\hbar^2\pi^2}{8ma^2} \qquad E_1 = \frac{\hbar^2\pi^2}{2ma^2}$

$u = \cos\frac{\pi x}{2a}$

c) $\mathcal{H}\psi = -\frac{\hbar^2}{2m}\psi'' + A\cos\frac{\pi x}{2a}\psi = E\psi$

d) $\Delta E_0 = \int_{-a}^{a} u_0^2\, A\cos\frac{\pi x}{2a}\,dx = \frac{A}{a}\int_{-a}^{a}\cos^3\frac{\pi x}{2a}\,dx = \frac{2A}{\pi}\int_{-1}^{1}(1-u^2)\,du$

$$\boxed{\Delta E_0 = \frac{8}{3}\frac{A}{\pi}}$$

$\Delta E_1 = \int_{-a}^{a} u_1^2\, A\cos\frac{\pi x}{2a}\,dx = \frac{A}{a}\int_{-a}^{a}\sin^2\frac{\pi x}{a}\cos\frac{\pi x}{2a}\,dx = \cancel{\frac{2a}{\pi}\frac{A}{a}}$

$$= \frac{A}{a}\cdot 4\cdot\frac{2a}{\pi}\int_{-1}^{1}u^2(1-u^2)\,du = \boxed{\frac{32}{15}\frac{A}{\pi} = \Delta E_1}$$